2103000203023001
EXAMINATION NOVEMBER 2024
BACHELOR OF SCIENCE (NON-NEP) (THIRD SEMESTER)
MTH-301-MATHEMATICS-V

[Time: As Per Schedule] **[Max. Marks:50]**

Instructions:
1. Fill up strictly the following details on your answer book
   a. Name of the Examination : **BACHELOR OF SCIENCE (NON-NEP) (THIRD SEMESTER)**
   b. Name of the Subject : **MTH-301-MATHEMATICS-V**
   c. Subject Code No : **2103000203023001**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.

Seat No:

Student's Signature

Q.1 Answer any five questions. **10**

(1) Find the limit of the function: $\lim_{(x,y)\to(2,1)}(x+2y)$

(2) If $x = rcos\theta, y = r\text{sine}\,\theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

(3) Examine whether f_{xy} and f_{yx} are equal or not for the function, $f(x,y) = xlog\ y + ylog\ x. x, y \in R^+$.

(4) Find $xu_x + yu_y$ where $u = x^2 + 3xy + 2y^2$.

(5) State the necessary condition for the function $f(x,y)$ to be maximum or minimum.

(6) If $\phi(x,y,z) = 2x^2y^3 - 3y^2z^3$ then find the value of grad ϕ at point (1, 1, 1).

(7) Define: Homogeneous function.

(8) If $\vec{r} = (1+t)\hat{\imath} + (t^2+t+1)\hat{\jmath} + (t^3+t^2+t+1)\hat{k}$ then find $\frac{d\vec{r}}{dt}$ and $\frac{d^2\vec{r}}{dt^2}$ at point $t = 0$.

Q.2 Answer any two questions. **10**

(1) Discuss the continuity of the function $f(x,y)$ at the point (1,1) where

$$f(x,y) = \frac{x^2+y^2}{x+y}; x+y \neq 0$$
$$= 1 \quad ; x+y = 0$$

(2) If $u = log(x^3 + y^3 + z^3 - 3xyz)$ then prove that

(i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

(ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 = \frac{-9}{(x+y+z)^2}$

(3) If $H = f(y-z, z-x, x-y)$ then prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$

Q.3 Answer any two questions. **10**

(1) If f is differentiable function in x and y such that $xf_x + yf_y = mf(x,y)$ then prove that f is homogeneous function of degree m.

(2) If $u^3 + v^3 = x + y,\ u^2 + v^2 = x^3 + y^3$ then show that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{1}{2}\frac{y^2-x^2}{2uv(u-v)}$.

(3) Expand $f(x,y) = \frac{y^2}{x^3}$ into the series expansion form up to the second order terms in the powers of $(x-1)$ and $(y-1)$.

Q.4 Answer any two questions. **10**

(1) Find the point $P(x,y)$ from which the sum of the squares of distances from the axes and the line $x + y - 8 = 0$ is minimum.

(2) Find the extreme value for the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.

(3) Discuss about the extreme points of the function $u = xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right)$.

Q.5 Answer any two questions. **10**

(1) If $\vec{v} = x^2yz\hat{\imath} + xy^2z\hat{\jmath} + xyz^2\hat{k}$ then find the value of div $(curl\vec{v})$.

(2) If $r = |\vec{r}|$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then show that
(a) $\nabla f(r) = f'(r)\nabla r$ and (b) $\nabla r^n = nr^{n-2}\cdot\vec{r}$.

(3) If $\vec{f} = 2xy\hat{\imath} + (x^2 - y^2)\hat{\jmath}$ and c is the curve $y^2 = x$ of arc at the points (0, 0) to (0, 1) then find $\int_c^{\vec{f}} \bullet\, d\vec{r}$.

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